

A Digital Image Watermarking Method Based on the Theory of Compressed Sensing

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Abstract

By combining the compressed sensing theory and the digital watermark technology, a digital watermarking method based on the theory of compressed sensing is proposed in this paper. The method is divided into three steps: compressed sensing compression process, compressed sensing recovery process and compressed sensing extraction. The watermark is embedded through the embedded mechanism between the compressed sensing compression process and the compressed sensing recovery process. The experiments show that the proposed watermark method has stronger robustness and is more safe to against the attacks.

Keywords

Digital Image Watermarking; Compressed Sensing; Sparse Transformation

Introduction

With the rapid development and widespread use of multimedia and network technologies, the transmission and acquisition of digital media become more and more convenient. Digital watermarking technology is an important digital copyright protection technique, which has been extensively studied in the research community and widely used in reality. Imperceptibility and robustness are two main requirements for the digital watermarking technology. In 2006, Candes mathematically proved that the original signal can be reconstructed accurately from part of the Fourier transform coefficients. Based on this theory, Candes and Donoho proposed the concept of "compressed sensing" [1,2]. The traditional signal sampling is based on Nyquist Theorem and the samples are taken with equal space. Unlike the traditional signal sampling, compressed sensing takes the sampling according to the characteristics of signals (e.g., the projection of the signal in some space has sparse property) and the sampling is not necessarily with equal space. To the best of our knowledge, compressed sensing technology has not been applied to digital watermarking so far. In this paper, we

propose a digital watermarking method based on compressed sensing technology. The model of the compressed sensing watermark technology is shown as follow:

As shown in figure 1, the compressed sensing watermark technology can be divided into three steps: compressed sensing compression process, compressed sensing recovery process and compressed sensing extraction. The compressed sensing compression process can further be divided into signal sparse transformation, sensing matrix construction, linear measurement. The compressed sensing recovery process basically refers to the compressed signal reconstruction. The compressed sensing extraction takes charge of watermarking extraction. The watermarking embedding is accomplished through the embedded mechanism, which is between the compressed sensing compression process and the compressed sensing recovery process. The watermark extraction is accomplished through the compression sensing extraction mechanism.

There are many advantages in compressed sensing technology. It can break through the Nyquist Theorem's sampling limit frequency. The sampling rate can be greatly decreased so that the compression ratio can be improved accordingly.

However, there is a necessary condition to apply the compressed sensing theory: the signal must be sparse. Signal is defined as sparse signal if only a small number of elements are non-zero [3,4]. But most of the natural images are not sparse signals. In order to apply the compressed sensing theory, we have to do the sparse transformation on the original image signals. The sparse transformation can be achieved by the following formula (1):

$$f = \sum_{i=1}^N x_i \varphi_i \quad \text{or} \quad f = \Phi x \quad (1)$$

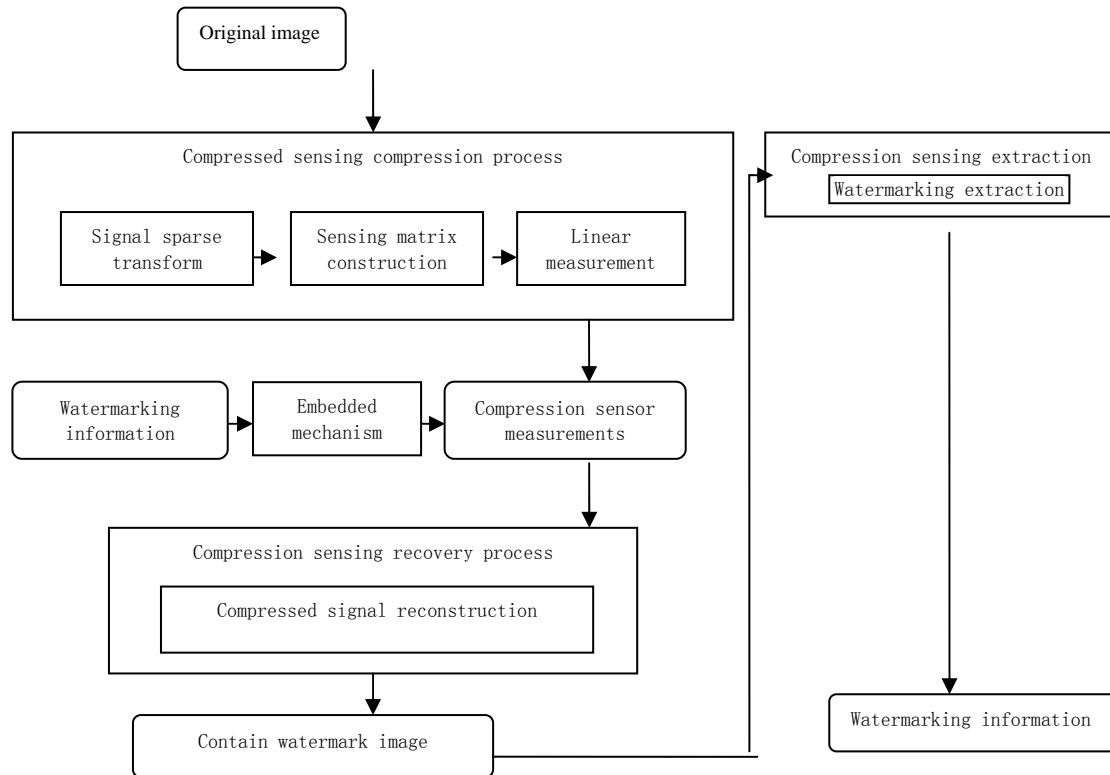


FIG. 1 COMPRESSED SENSING WATERMARK TECHNOLOGY MODEL

In formula (1), ϕ is the sparse representation base, which is a vector with dimension $N \times 1$. f represents the original image signal. X represents the sparse signal of f after the sparse transformation. The most commonly used sparse transform techniques include: the wavelet transform, the discrete cosine transform, and the discrete Fourier transform. In this paper, based on the basic idea of the wavelet transform, we propose a sparse transform method. The basic idea is to construct orthogonal transformation matrix. The sparse transformation is accomplished by the matrix diagonalization similar way, i.e., let the original image matrix pre-multiply the orthogonal matrix and post-multiply the transposed matrix of the orthogonal matrix). This way can obtain considerable sparse degrees. Moreover, the transformation is an reversible process, which is very convenient in the application of digital watermarking. The method is shown as follows:

Sparse transformation:

Sparse signal = orthogonal matrix \times original image \times orthogonal matrix T (2)

Inverse sparse transformation:

The original image = orthogonal matrix $T^T \times$ sparse signal \times orthogonal matrix (3)

Through this kind of transformation and the reverse

transformation mechanism, the discussion of the compressed sensing digital watermarking technique has been transferred from the original image to sparse signal domain. The following discussion of the compressed sensing watermarking technology will be continued from the sparse signal processing.

Sensing Matrix Structure

In the compressed sensing compression process, sensing matrix [7] construction is a very important part. It determines whether the compressed sensing compression process can be implemented. Moreover, whether the sensing matrix is properly selected goes to whether or not the compression will be achieved and whether or not the signal can be reconstructed accurately. The formula (4) illustrates how to construct the sensing matrix.

$$\text{Measured value} = \phi \psi \text{ Original signal} \quad (4)$$

The formula (4) (ψ is the original signal) is equivalent to the sparse transformation as has been mentioned above. Here ψ represents the sparse transform representation base, which can be approximately considered as the orthogonal matrix for the sparse transformation. ϕ is the observation matrix. We define the result of the product of $\phi \psi$ (which is denoted as

ϕ') as the sensing matrix. Sensing matrix is the core of the compressed sensing. The construction of sensing matrix is very important, which also needs to satisfy some conditions.

Candes and Tao ^[5] proposed and proved the condition that the sensing matrix needs to satisfy, i.e., the restricted isometry property. For any K sparse signal x , it requires that:

$$(1 - H_k) M / N \|x\|_2^2 \leq \|\phi'x\|_2^2 \leq (1 + H_k) M / N \|x\|_2^2 \quad (5)$$

H_k is a constant between (0, 1). M , N are the dimensions of the matrix. If the sensing matrix satisfies (5), we say the sensing matrix satisfies the restricted isometry condition. This condition means that as long as the sensing matrix meets the restricted isometry condition, the N dimension signal can be reconstructed accurately according to the K ($N > K$) vectors. In brief, the restricted isometry condition requires that the observation matrix ϕ and sparse representation base ψ are independent. It means that the row vectors of the observation matrix cannot be represented by the column vectors of the sparse representation base. Similarly, the column vectors of the sparse representation base cannot be represented by the row vectors of the observation matrix.

In the compressed sensing watermark processing flow, if the original image is compressible (i.e., the original image has approximate sparse representations in some linear transformations), there will contain enough information to reconstruct the signal information by only doing a small number of projections for the original image. As has been mentioned, the construction of the sensing matrix is the most important part in the compressed sensing watermarking method. In the sensing matrix, the signal observation matrix and sparse representation base must be highly independent. In other words, only when they are independent, it can ensure that the measured values from the observation matrix can be reconstructed to the original signal with high quality. Otherwise, if they are dependent, the dimension of the sensing matrix will be decreased due to the linear dependency. In this manner, the number of useful measured values will be reduced so the requirements to reconstruct the original signal cannot be reached. Many matrices can be used as the observation matrix to make sure the sensing matrix can satisfy the restricted isometry condition, such as the consistent

ball matrix, the binary random matrix, the partial Fourier matrix, the partial Hadamard matrix, and Toelitz matrix etc.

In this paper, we take the binary random matrix as the observation matrix. The reason is that the binary random matrix can satisfy the restricted isometry condition as well as it makes the linear measurement process become more easy and simple. Moreover, the efficiency of the binary random matrix is much higher than other matrices that satisfy the restricted isometry condition.

Linear Measurement

In the definition of sensing matrix, the observation matrix is used (see formula (4)). In this section, the observation matrix will be used to do the linear measurement. The observation matrix ^[6] is the matrix that is used to measure the measured values (i.e., the compressed signal values) in the compressed sensing process. Generally, the size of the observation matrix determines the size of the measured value as well as the compression degree in the compressed sensing process. A small size observation matrix can achieve a high compression ratio. However, the quality of the reconstruction will be decreased accordingly. Conversely, a large size observation matrix can lead to a high quality reconstruction. However, the compression ratio will be decreased. According to the experience, for the large size texture images, the most appropriate size of the observation matrix to achieve a high quality reconstruction is 2/3 size of the original image. For the simple scenery pictures, 1/2 size of the original image can still achieve high quality reconstruction.

In this paper, the linear measurement is carried out according to formula (6) ^[4]

$$\text{Measured value} = \text{Observation matrix} \times \text{Sparse signal} \quad (6)$$

According to formula (6), if the observation matrix has $M \times N$ ($M < N$) dimensions and the sparse signal has $N \times N$ dimensions, the measured value has $M \times N$ dimensions. The size of the measured value is much smaller than the size of the sparse signal, which conforms the compression function of the compressed sensing technology. The measuring method in formula (6) can meet the basic requirements of the compressed sensing theory. Moreover, the measurement is very simple and highly efficient. So far, we get the measured value of the compressed sensing

compression process. Based on the measured values, the watermarking embedding mechanism can be implemented, which will be presented in next section.

The details of the watermarking embedding mechanism are as follows: (1) read the watermark information. (2) choose the position where to embed the watermarking in the measured values (here we embed the watermarking with equal space). (3) check the highest bit of the measured value in this position to see whether the parity satisfies the embedding requirement (0 represents even and 1 represents odd in the watermarking). If the parity is satisfied, go on to embed next bit of the watermarking. Otherwise, check the relationship between the highest bit and a threshold. If it is smaller than the threshold, the highest bit is increased by 1, otherwise, it is decreased by 1. (4) repeat steps (1)-(3) until the watermarking is completely embedded.

The Experimental Results

In the compressed sensing digital watermarking technique, the watermark is embedded in the measured values. The watermarking information is noise of the measured value, which will influence the quality of the image reconstruction. So, in order to control the influences, reasonable watermark information should be selected to embed. In the experiments, we embed different amount of watermarking information to analyse the impact on the image reconstruction.



(a) 64 x 64 pixel embedded watermark



(b) embedded watermark 85 * 85 pixels



(c) 100 x 100 pixel embedded watermark



(d) embedded watermark 128 * 128 pixels

FIG. 2 ANALYSIS IMAGE OF CAPACITY EXPERIMENT

In this experiment, the size of our measured matrix is 190×256 . The size of the watermarking information is set to $1/7$, $1/10$, $1/5$ size of the measured value matrix in respectively to analyse the impact on the image reconstruction. The source image of the watermark is the binary image lena.bmp. We use different resolutions of this picture to create different size watermarking information to embed. In our experiments, four resolutions are used, which are 64×64 , 85×85 , 100×100 and 128×128 . The results of the reconstruction after embedding the watermark information are shown in figure 3.

As can be seen in figure 3(a), when the resolution is 64×64 (which is about $1/10$ of the measured value matrix), the impact of the watermark information on the reconstruction is insignificant. When the resolution is set at 85×85 (which is about $1/7$ size of the measured value matrix), the quality of the reconstruction is still good enough, which can be seen from figure 3(b). However, when the resolution is set to 100×100 (which is about $1/5$ size of the measured value matrix), the influence of the watermark information on the reconstruction is significant. As shown in figure 3(c), the quality of the reconstruction is greatly decreased and there are a lot of ripples in the reconstructed image. When the resolution is set to 128×128 , the reconstruction is greatly affected by the watermark information. As shown in figure 3(d), it is

already difficult to recognize the reconstructed image.

From the results we can see that, the most appropriate size of the watermark information is about 1/7 size of the measured value matrix. When the watermark information is embedded with this size, the quality of the reconstruction is satisfactory.

TABLE 1 GAUSSIAN NOISE TYPE

Image	a	b	c	d
Gaussian noise mean	0	0	0	0
Gaussian noise variance	0.001	0.01	0.05	0.1

We also examined the performance of the compressed sensing watermark technology on the geometric attacks.



(a) Level flip including watermark image



(b) Extracted watermark image

FIG. 4 HORIZONTAL FLIP EXPERIMENT RESULTS.

To contain the watermark image of vertical flip experiment.



(a) Vertical flip including watermark image



(b) Extracted watermark image

FIG. 5 THE EXPERIMENTAL RESULT OF VERTICAL FLIP.

Figure 4 shows the results of the horizontal flip experiment. We can see that the proposed compressed sensing digital watermark method can resist the horizontal flip. The watermark image can be extracted successfully. However, from figure 5 we can see that, the compressed sensing digital watermark method cannot resist the vertical flip. This is because we use the product of the matrix in the current measurement method. The measured values are not affected when the positions of the column vectors of the matrix are changed in horizontal flip. Only the positions of the measured values are changed as the change of the column vectors. However, the measured value will be affected in vertical flip. So, the correct watermark cannot be extracted. Therefore, the compressed sensing watermark method cannot resist vertical geometric attacks.

Conclusions

The paper proposed a digital watermarking method which is based on the theory of compressed sensing. The method is divided into three steps: compressed sensing compression process, compressed sensing recovery process and compressed sensing extraction. The method was evaluated by the experiments. The experiments showed that the embedded watermark has stronger robustness and is more safe to against the attacks. Currently, it still cannot resist vertical geometric attacks, which can be considered in our future work.

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